PARAMETERIZATION OF THE MIXING RATIO VERTICAL DISTRIBUTION BY A POWER-LAW PROFILE

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THESIS

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Parameterization of the Mixing Ratio Vertical Distribution by a Power-law Profile

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ABSTRACT

An investigation is made into the accuracy of the description of the actual mixing-ratio profile by a power-law approximation. The approximation is obtained by a least-squares technique in order to derive the best-fit exponent λ of the pressure profile. The λ-values were computed for soundings at 12 geographically diverse stations over the United States during the period 16-23 March, 1971. The λ-values at each station were found to undergo a time-variation based mainly on the synoptic-scale variations in the mixing-ratio profiles. A stepwise multiple regression procedure involving up to four variables from the temperature-humidity soundings was utilized in order to "predict" the value of λ from gross-parameters of the soundings by station. Tests were performed at four stations in comparing the observed precipitable water vapor W_o with that computed from the power-profile W_{cs} which depended upon λ computed from the sounding. The values of W_{cs} gave a high correlation, $R(W_{cs}, W_o) \geq 0.97$, for each of the four stations. However, the values of W_{cm} using λ from the predictive multiple regression equation gave somewhat larger standard errors at each of the four stations. The W_{cm}-values when compared to W_o indicated some measure of predictive skill.

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TABLE OF SYMBOLS AND ABBREVIATIONS

A_i	Regression coefficients of the ith variable added
e	Vapor pressure
ϵ_i	Residual (or error) after ith variable added
λ	Least-squares pressure ratio exponent
λ_{cs}	Least-squares pressure ratio exponent from power-profile technique
λ_{cm}	Least-squares pressure ratio exponent from multiple regression technique
σ_X	Standard deviation of variable X
$\sigma_t(\lambda)$	Temporal standard deviation of exponent
F	F - statistic for statistical confidence
FNWC	Fleet Numerical Weather Central
GMT	Greenwich Mean Time
mb	Millibars
N	Number of observations or size of data-sample
P_{top}	Pressure at the top of sounding
P_s	Pressure at the surface
R_{12}	Correlation coefficient of dependent variable Y to independent variable X
R_m	Multiple regression correlation coefficient
R_{12}^2	Fractional explained variance
RH	Relative humidity
$S_{Y\|X}$	Standard error of Y after use of predictor X
S.S.	Sum-of-squares
W	Precipitable water vapor

W_o	Observed precipitable water vapor
W_{cs}	Precipitable water vapor calculated using $_{cs}$
W_{cm}	Precipitable water vapor calculated using $_{cm}$
w	Mixing ratio
w_{500}	Mixing ratio at 500 mb
X	Independent variable
Y	Dependent variable

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I. INTRODUCTION

Reitan (1963) established a correlation factor of 0.98 between the logarithm of the monthly precipitable water vapor and monthly dewpoints based on 540 observations at numerous locations. The regression formula used by Reitan was

$$\ln W = A + B\, t_d \tag{1}$$

where A = -0.981 and B = 0.0341 are the best fit coefficients resulting from the regression procedure.

Smith (1966) has shown that an equation of form (1) may be derived from the well known empirical equation of Tetens (1930)

$$e = E_o \times 10^{(a\, t_d - B)/(t_d + \gamma)} \tag{2}$$

provided that the mixing-ratio profile may be parameterized. The constants are E_o = 6108 dynes cm^{-2}, a = 7.5, B = 238.1 F, and γ = 395.1 F. The required parameterization of the mixing-ratio can be accomplished with reasonable accuracy by the power law approximation

$$w = w_o\, (p/p_o)^{\lambda} \tag{3}$$

By combining equations (2) and (3) with

$$w_o = \varepsilon\, (e_o/p_o) \tag{4}$$

Smith was able to show that

$$\ln W = \ln \varepsilon E_o/g + 2.3026\, (a\, t_d - B)/440 - \ldots$$
$$- \ldots - \ln(\lambda + 1) \tag{5}$$

Since climatological values of the exponent λ are variable with latitude, Smith concluded that the coefficient A of (1) is likewise dependent upon latitude in Reitan's formulation.

TABLE I

λ -values from Smith (1966)

LATITUDINAL ZONE -^{o}N	WINTER	SPRING	SUMMER	FALL	ANNUAL AVERAGE
0 - 10	3.37	2.85	2.80	2.64	2.91
10 - 20	2.99	3.02	2.70	2.93	2.91
20 - 30	3.60	3.00	2.98	2.93	3.21
30 - 40	3.04	3.11	2.92	2.94	3.00
40 - 50	2.70	2.95	2.77	2.71	2.78
50 - 60	2.52	3.07	2.67	2.93	2.79
60 - 70	1.76	2.69	2.61	2.61	2.41
70 - 80	1.60	1.67	2.24	2.63	2.03
80 - 90	1.11	1.44	1.94	2.02	1.62
NORTHERN HEMISPHERE AVERAGE	2.52	2.64	2.62	2.70	2.61

Bolsenga has attempted to adapt Reitan's formulation (1) to daily and even to hourly observations and finds that the explained variance of total water vapor falls off to 72% and 64% in the respective cases. This could have been anticipated since total water vapor depends in each case upon the entire mixing-ratio profile, which depends less upon surface dewpoint when a decreased time interval of sampling is employed.

The equivalence of equations (1) and (2) to the λ-profile (3) holds best, according to Smith, when applied to climatological data. Climatological values of λ were calculated by Smith (1966), from London's global mean radiosonde data (1957), with resultant λ-values presented as a function of latitude and season as shown in Table I.

The concept of an accurate mixing-ratio moisture profile from soundings composed of few or widely spaced levels is a very valuable tool. It is especially important in the field of radiation flux measurement where values for the mixing-ratio (moisture profile description) are needed at all levels to the top of the atmosphere but are frequently not available from sounding levels much above 500 mb. Recently Plante (1973) and Martin (1973) have extended the use of Smith's power-law profile to one of the form

$$w = w_{500}(p/500)^{\lambda} \tag{6}$$

with the reference or key level at 500 mb rather than a surface reference level which has a diurnally variable mixing ratio. Martin (1973) using the Fleet Numerical Weather Central, Monterey, California, computer printout of mixing ratio at six pressure levels in the vertical at FNWC grid points found a correlation coefficient of .998 or higher between observed mixing-ratio and a mixing-ratio computed by Eq. (6), based on λ-values calculated at each grid point for the 75 grid points considered. Martin

concluded that the power-fit as described by Eq. (6) is accurate especially with soundings having widely separated reporting levels in the vertical.

In this thesis the following questions will be investigated: for a given station for a given period of time, can a power-fit of the nature of (6) be obtained which is representative of the actual moisture profile at 50 mb-level increments, and what degree of accuracy can be expected? Further, the conditions which would tend to invalidate the accuracy of the method will be delineated. Two methods of verification of the results will be considered:

(1) the explained variance of mixing-ratio resulting from the use of power-law profile, and

(2) the accuracy for the calculated precipitable water obtained from the profile-parameterization compared to that observed.

II. DATA PROCESSING

A. THE ORIGINAL DATA

For the water vapor analysis, twelve stations were selected on the basis of their locations and the availability of data. The time chosen was an eight day period from 16 March to 24 March, 1971. The station locations, elevations, and mean surface pressures are shown in Table II. As can be seen from Figure 1, the stations selected lie in a band across the southern, and in a second band across the northern continental United States. The data was obtained from NWSED, Ashville, North Carolina. All data levels were selected at levels corresponding to integral multiples of 50 mb (up to 400 mb), in addition to the surface level.

The following analysis was used as the basis of solving for λ , starting from Eq. (6):

$$\log \frac{w(p)}{w_{500}} = \lambda \log \frac{p}{500} \tag{7}$$

Here p ranged from the surface value to p = 400 mb, and corresponding values w(p) were computed at each level p using sounding-level data. If the left side of Eq. (7) is denoted as Y, then this equation becomes

$$Y = \lambda X \tag{8}$$

where $X = \log (p/500)$. In the average sounding which extends to p = 1000 mb, there would be 14 simultaneous values (Y_i, X_i) so that only a best-fit solution for λ is feasible.

This amounts to solving the linear regression equation, Eq. (8), for the coefficient λ of the independent variable, with the condition of the zero-intercept forced on the constant term. In the program library of the

TABLE II

Information on stations selected

STATION	INDEX NUMBER	^{o}N LATITUDE	^{o}W LONGITUDE	(FEET) ELEVATION	MEAN PRESSURE AT SURFACE
Charleston	72208	32 - 54	80 - 02	48	1012 mb
Waycross	72213	31 - 15	82 - 24	142	1011 mb
Lake Charles	72240	30 - 07	93 - 13	32	1012 mb
El Paso	72270	31 - 48	106 - 24	3916	877 mb
San Diego	72290	32 - 44	117 - 10	28	1013 mb
Caribou	72712	46 - 52	68 - 01	628	990 mb
Maniwaki	72722	46 - 22	75 - 59	559	992 mb
Sault Ste. Marie	72734	46 - 28	84 - 22	724	987 mb
Bismarck	72764	46 - 46	100 - 45	1660	953 mb
Glasgow	72768	48 - 13	106 - 37	2298	931 mb
Spokane	72785	47 - 38	117 - 32	2365	930 mb
Quillayute	72797	47 - 27	112 - 18	450	996 mb

Figure 1: Locations of selected stations.

W. R. Church Computer Center, the most convenient program for performing this objective was BIMED 02R (Dixon, 1966). While this program is primarily used for stepwise regression in the case of multivariate regression analysis, it also has the desirable option of making the required transformations of variables first to ratios, and then to their logs as in Eq. (7). This requires that the sounding data has been prepared in properly formatted form on IBM cards.

B. CALCULATION OF MIXING-RATIOS AND OF λ

Having decided on the method of handling the regression, the next computation was that of mixing-ratio to be specified by the power-profile fit, Eq. (6).

The data as it was received (by courtesy of Mr. Russell Schwanz), was card-punched in a form containing the pressure, temperature (C), and relative humidity at 50 mb-intervals from the surface to the top of the moisture sounding (which was generally at 400 mb). The next step then was to compute the mixing-ratio in its approximate form

$$w = \frac{.62197\ (RH)\ e_s\ (T)}{p} \qquad (9)$$

where e_s is the saturation vapor pressure computed over a plane water surface for $T \geq 0$ C, and e_s is computed with respect to equilibrium over a pure, plane ice surface at temperatures $T < 0$ C. In doing this at each sounding level, the two versions of the Goff-Gratch equations for e_{sw} and e_{si}, respectively, were used in the forms indicated below as Eq. (10) and Eq. (11).

$$\log_{10} e_{sw} = -7.90298\ \left(\frac{T_s}{T} -1\right) + 5.02808\ \log_{10}\ \left(\frac{T_s}{T}\right) - \ldots \qquad (10)$$

$$\ldots -1.3816 \times 10^{-7}\ (10^{11.334(1-T/T_s)} -1) + \ldots$$

$$\ldots +8.1328 \times (10^{-3.49149(T_s/T-1)} -1) + \log_{10} e_{ws}$$

$$\log_{10} e_{si} = -9.09718(T_o/T-1)-3.56654 \log_{10}(T_o/T) + \ldots \qquad (11)$$
$$\ldots +0.876793(1-T/T_o) + \log_{10} e_{io}$$

where T_s = 373.16 K and T_o = 273.16 K.

In (10) e_{sw} is the saturation vapor-pressure of pure water at the steam-point temperature and e_{si} is that of pure water at the frost-point temperature (T < 0 C). Then w(p) is evaluated from either equation (10) or (11) utilizing Eq. (9). Once the values for w were obtained at each level in each sounding the best-fit value for λ was obtained in the manner suggested in connection with Eq. (8) and Section III (A). Thus a unique value of λ resulted for each sounding. The λ-values for each station were then grouped with other synoptic sounding data for each individual sounding in order to account for the reduced variance of λ for the sounding-sample for the particular station. In this connection, the stepwise regression program BIMED 02R was used to determine the explained variance of λ at each station. Comparisons were made of computed w-values resulting from Eq. (6) versus observed w-values over the sounding station.

C. CALCULATION OF PRECIPITABLE WATER VAPOR

Precipitable water was obtained by integration for both the observed soundings and for the corresponding λ-profile cases and a comparison was made for several stations. The formulas used in the calculations are shown as follows.

a. The observed value of precipitable water vapor is

$$W = \int_{P_1}^{P_{14}} \frac{w}{g}\, dp = \frac{1}{980}\left[\sum \frac{w_1 + w_2}{2} \Delta_{1.5} P + \frac{w_2 + w_3}{2} \Delta_{2.5} P + \ldots \right. \qquad (12)$$

$$\ldots + \frac{w_{12} + w_{13}}{2} \Delta_{12.5} P + \frac{w_{13} + w_{14}}{2} \Delta_{13.5} P \Big]$$

b. The model value of precipitable water vapor is

$$W = \int_{P_{top}}^{P_s} w_{500} \left(\frac{p}{500}\right)^{\lambda} \frac{1}{g}\, dp = \frac{w_{500}\ (500)}{g\ (\lambda + 1)} \left[\left(\frac{p}{500}\right)^{\lambda+1} \Bigg|_{400}^{P_s} \right] \tag{13}$$

$$= \frac{w_{500}\ (500)}{980\ (\lambda + 1)} \left[\left(\frac{P_s}{500}\right)^{\lambda+1} - \left(\frac{400}{500}\right)^{\lambda+1} \right]$$

The comparison of the sample sets of the two precipitable water values will be used as a second measure to determine the accuracy of the λ - profile method.

III. STATISTICAL CONCEPTS

A. THE LEAST SQUARES FIT

In this thesis, the least-squares fit was employed in at least two contexts. First, it was employed in solving Eq. (8) for λ at each station and sounding time. Here the variables Y and X are given pointwise by

$$X_i = \log \frac{P}{500} \text{ and } Y_i = \log \frac{w_i}{w_{500}} \qquad (14), (15)$$

with i representing a level numbered from one at the surface, to two at the next higher level (where P first becomes an integral multiple of 50 mb), etc. Standard texts in statistics (e. g. Brownlee, 1960) show that the parameter λ of the power-profile as in Eq. (8) can be deduced from the best-fit condition

$$\lambda = \frac{\sum X_i Y_i}{\sum X_i^2} \qquad (16)$$

If the zero-intercept had not been required in (16), λ would have been estimated from

$$\lambda = \frac{1}{N} \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sigma_x^2} \qquad (17)$$

and the constant term of the best-fit simple regression would then be

$$A = \frac{\sum_{i=1}^{N} Y_i}{N} \qquad (18)$$

where N is the number of sounding-level observations entering into the power-profile (normally here N = 14).

The combination of conditions (17) and (18) gives the standard best-fit solution for the simple independent variable case

$$Y = A + \lambda X \tag{19}$$

Equation (16) gives the corresponding result when the zero-intercept ($Y_i = X_i = 0$) is required. In either case λ is related to the simple correlation coefficient R_{12} and the variables Y and X through the standard formula

$$R_{12} = \lambda \frac{\sigma_X}{\sigma_Y} \tag{20}$$

Hence a value of $R_{Y|X}$ close to unity means an effective least-squares fit by the power-profile and simultaneously a reliable value of λ . In Eq. (20), the statistical parameters σ_Y and σ_X are defined as

$$\begin{aligned} &\text{a.} \quad \sigma_Y = \left(\frac{Y_1^2 + Y_2^2 + \ldots Y_N^2}{N}\right)^{\frac{1}{2}} \\ &\text{b.} \quad \sigma_X = \left(\frac{X_1^2 + X_2^2 + \ldots X_N^2}{N}\right)^{\frac{1}{2}} \end{aligned} \tag{21}$$

Note that σ_Y and σ_X of (21) are applicable to the zero -intercept case and apply to the more general case of the standard regression formula

$$Y_i - \bar{Y} = R_{12} \frac{\sigma_Y}{\sigma_X} \left(X_i - \bar{X}\right) \tag{22}$$

when Y_i is replaced by $Y_i - \bar{Y}$ and X_i is replaced by $X_i - \bar{X}$.

It was possible to apply BIMED 02R regression procedure to the 14-values of X_i and Y_i per sounding (that is, with each sounding listed on 14 cards) and to generate not only the λ -value for that sounding, but also the simple correlation R_{12}, which is a measure of the "goodness of fit" of the power-profile for the particular sounding case. Such

results for λ and R_{12} have been tabulated under each station and for each observation time (Tables III through XIV).

B. THE STEPWISE REGRESSION PROCEDURE

If there are N samples of the statistically related variables $(Y_{1i}, X_{2i}, X_{3i}, \ldots, X_{ki})$ $i = 1, \ldots, N$, it is possible to apply the least-squares concept to minimize the least-squares form

$$\sum_{i=1}^{k} (Y_{1i} - A_1 - A_2X_{2i} - \ldots - A_kX_{ki})^2 = \text{minimum} \qquad (23)$$

The procedure of selecting the least-squares fit of coefficients to minimize Eq. (23) is called multiple linear regression. In this process, there is no assurance that the variables $X_2, \ldots, X_k$ used to explain the variation of Y_1 are the most effective, and, in fact, some of the X_i may not contribute to the explained variance in any significant manner.

BIMED 02R is a statistical procedure for developing a multiple regression equation in which the variable X_i added at each step accomplishes the greatest reduction in the unexplained sum-of-squares, $\sum (Y_i - Y)^2$. If k-1 variables are to be added in a stepwise manner the first one added is that which has the largest simple correlation coefficient R_{12} of all k-1 possible comparisons. As a result, the first estimate to the desired multiple regression equation may be obtained in the form:

$$Y_1 = a_{11} + a_{12}X_2 \qquad (24)$$

with a_{11}, a_{12} selected as in (17), (18). A property of this first-step selection is that the sum-of-squares explained by Eq. (24) is given by

$$(\text{S.S. Expl})_1 = R_{12}^2 \left[(Y_{11} - \bar{Y}_1)^2 + \ldots + (Y_{1N} - \bar{Y}_1)^2 \right] \qquad (25)$$

A result which is proved in standard statistics texts (e.g. Crow et al, 1960).

The BIMED 02R program then forms the data-sample residuals. After one step, all values have the form

$$Y_{1i}-a_{11}-a_{12}X_{2i} = Y_{1i}^{(1)} \tag{26}$$

The program then examines all of the simple correlations of $Y_1^{(1)}$ on $X_3,\ldots,$ X_k (the remaining independent variables) and selects that which has the highest simple correlation coefficient among $R(Y_1^{(1)},X_i)$, $i = 3,\ldots,k$. Since this simple correlation is based upon the residual after the effect of variable X_2 has been removed (see Eq. (26)), it is typically called the first order partial correlation with the notation $R_{13.2}$, the understanding being that the variable having this highest correlation at step 2 and thus being selected is called X_3. Then the regression equation relating $Y_1^{(1)}$ to X_3 is obtained as described in connection with Eqs. (21), (22), (23) and assumes the form

$$Y_1^{(1)} = b_{11} + b_{13}\, X_3 \tag{27}$$

As a result of the second-step simple regression, Eq. (27), the sum-of-squares of Y_1 (not $Y_1^{(1)}$) is now statistically explained in the amount

$$(\text{SS Expl})_{1,2} = R^2_{1.23}\left[(Y_{11}-\bar{Y}_1)^2 + \ldots + (Y_{1N}-\bar{Y}_1)^2\right] \Big/ N-1 \tag{28}$$

The statistical parameter $R^2_{1.23}$ is the so called multiple correlation at step 2, that is to say of variable Y_1 depending jointly on the variations of both X_2 and X_3. These independent variables were selected according to the criteria of having the largest simple correlation, and the largest first-order partial correlations respectively.

The multiple correlation coefficient satisfies the important chain-rule of multiplication

$$1 - R^2_{1.23} = (1-R^2_{12})\,(1-R^2_{13.2}) \tag{29}$$

and after k-1 steps, this rule assumes the form

$$1 - R^2_{1.23\cdots k} = (1 - R^2_{12})\,(1 - R^2_{13.2\cdots k})\cdots(1 - R^2_{1k.23\cdots k-1}) \quad (30)$$

Just as $R^2_{1.23}$ describes the fractional explained sum-of-squares of Y in Eq. (28), so $1-R^2_{1.23}$ describes the fractional unexplained sum-of-squares about the regression surface. This parameter has decreased relative to that of step one, provided $R^2_{1.23} > R^2_{12}$. The standard error of estimate is related to the multiple correlation coefficient by

$$S^2_{Y|X} = \sigma^2_Y\,(1-R^2_{12}) \quad (31)$$

The parameter standard error, $S_{Y|X}$, will be listed in Tables III,...,XIV. In (31), the subscript Y|X denotes the best fit of Y on all X_i.

The most attractive feature of the stepwise regression is embodied in the chain-rule of Eqs. (29), (30), with each step making the selection of the highest partial correlation coefficient for use in Eq. (30). The result is that when the unexplained fractional sum-of-squares does not become significantly smaller, the process of adding variables to the right-hand side of

$$Y_1 = A_1 + A_2X_2 + A_3\,X_3 + \ldots\, A_kX_k \quad (32)$$

should be terminated at the (k-1)th step.

In Section IV, application of BIMED 02R will be used on the 16-sample values of λ derived at each individual station. This is done since it turns out that a time variation in λ occurs with the synoptic situation at individual stations. In some cases this variation, as indicated by the standard deviation σ_λ, is so small as to be limited by the accuracy of the data. In other cases, a rather large variation of λ occurs and it was desired to consider how the time-varying structure of the soundings contribute to the λ -variation. For this further study each λ -value

was added on an IBM card containing certain relevent parameters of the sounding from which λ was derived. For all soundings and stations a set of six independent parameters (for example T(850) and w(850) and others to be described in Section IV) were included along with the λ - value for the identical sounding.

Thus BIMED 02R was also used in the important stepwise sense in describing the λ -variation in the sounding structure over the set of the 16 soundings per station. The detailed discussion of the variables used in the regression for λ is given in Section IV (B, C), together with the discussion of the results.

C. ESTIMATES OF STATISTICAL SIGNIFICANCE

1. Of λ -Profiles at Sounding Stations

Here the correlation coefficient R_{12} of the regression equation (7) is the basis of the estimation of the accuracy of the λ -profile. The significance of the test is based upon Eq. (25) which gives the result

$$\text{Fractional Explained Variance} = R_{12}^2 \qquad (33)$$

whereas the fraction residual (or unexplained) variance is given by

$$\text{Fractional Unexplained Variance} = 1-R_{12}^2 \qquad (34)$$

The significance test is based upon the F-statistic (which is listed in the BIMED 02R output)

$$F(1,\ 13) = \frac{R_{12}^2}{(1-R_{12}^2)/13} \qquad (35)$$

The critical value of F(1,13) at the 99% confidence level is listed in statistical tables as

$$F_{.01}(1,13) = 9.07$$

This 99% confidence level is satisfied whenever R_{12} of Eq. (34) exceeds

$$R_{12} \text{ (critical) } \geq .641$$

In all but one of the λ-profiles, R_{12} exceeded this value, and in this exception R_{12} was 0.614, corresponding to a confidence level of 97.5%.

In summary, the statistic R_{12}^2 listed for each sounding at the 12 stations, is the explained fraction of the variance of Y in that sounding (see Tables III,..., XIV).

Finally, BIMED 02R lists also the standard error of the coefficient λ

$$\text{Standard Error of } \lambda = \lambda \sqrt{\frac{1-R_{12}^2}{R_{12}^2(N-1)}} \quad ; N = 14 \tag{36}$$

This statistic appears in column 3 of Tables III,... XIV, and describes the sharpness of the λ-fit in Eq. (7).

2. Significance Tests of the Temporal Variation of λ

Here the fractional explained variance may be computed from Eq. (28) after the kth entry of an independent variable X_k drawn from the synoptic sounding variables has been included in

$$\lambda = A_o + A_1X_1 + \ldots + A_kX_k \tag{37}$$

The fractional explained variance is given by R^2 in accordance with (28), whereas the fractional residual variance is $1-R^2$. In general when $\sigma(\lambda)$ is sizeable, it is possible to generate a multiple regression equation of form (37), which significantly reduces the residual variance. In general the criterion used for entering additional variables for explaining the λ-variance was based on a stepwise selection which added at least 2% to the total explained fractional variance. This strategy was used because the F-statistic for this case is

$$F_k\ (1,\ N\text{-}k\text{-}1) = \frac{R_k^{\ 2} - R_{k-1}^{\ \ 2}}{(1 - R_k^{\ 2})/(N - k - 1)} \tag{38}$$

and here the critical value of F_k is difficult to surpass with N = 16 observations. The added fractional explained variance at step k is identical to the numerator of Eq. (38), and is required to be at least 2% at each step.

IV. RESULTS

A. DISCUSSION OF λ -PROFILES

This particular phase of the research began with computer cards containing the following individual sounding data: pressure (mb), temperature (K), and mixing ratio (gm/kg). There were 16 soundings of up to 14 levels at each of the 12 stations (Table II).

The BIMED 02R program was utilized in a least-squares context to solve Eqs. 7 and 16 for a best fit λ for each sounding at each station. The "zero-intercept" was chosen in the correlation so that the computer solution to Eq. (7) was of the form of Eq. (8) instead of Eq. (19). A discussion of the statistical methods and parameters is given in Section III (A). Table III lists the best-fit λ -value for each available sounding at Waycross, Ga., over the test period.

It should be noted that the least-square procedure for determining from

$$Y_i = \lambda X_i + \varepsilon_i \tag{39}$$

minimizes the sum-of-squares of the residuals $\sum \varepsilon_i^2$. Hence the estimator Y may be defined as

$$Y = \lambda \log P/500 = \log \frac{w_c(P)}{w_{500}} \tag{40}$$

where the observed w_{500} appears in the denominator of Eq. (40) by the zero-intercept concept. On the other hand, the observed Y is defined as $Y = \log \dot{w}/w_{500}$ so that the residual becomes

$$\varepsilon_i = \hat{Y} - Y = \log \frac{w_c(P)}{w(P)} \tag{41}$$

In Table III, successive values of λ obtained by λ -profile fit [Eq. (39)] have been listed along with the standard error of each λ determination. The small standard error of λ indicates the close fit of the λ values by Eq. (16).

In addition, the fractional explained variance R_{12}^2 for each sounding is listed alongside R_{12} under column 4. Finally, the standard error of estimate is listed in the final tabular column as defined by Eq. (31). The value of $S_{Y|X}$ is related to the standard deviation σ_Y through

$$S_{Y|X} = (1 - R_{12}^2)^{\frac{1}{2}} \sigma_Y \tag{42}$$

These ratios of Eq. (42) obtained by applying the values of $S_{Y|X}$ and from columns 5 and 6 of Tables III through XIV are generally of the order of 0.3 or lower. There are exceptions, but these can be identified by the values of R_{12} (generally $R_{12} \geq .95$) for the respective soundings.

In Table III, the standard error may be written in terms of the mean residuals - squared as

$$S_{Y|X} = \left[\sum_{i=1}^{N} \left(\log \frac{w_c}{w} \right)_i^2 \Big/ 13 \right]^{\frac{1}{2}} = \log \frac{w + |\Delta w_c|}{w} \tag{43}$$

where the final term in Eq. (43) represents the mean scatter about the graph of $\hat{Y} = \lambda X$.

For example, at 0000 GMT, 21 March, 1971, the standard error at Waycross, Ga., was 0.1131. Therefore, $.1131 = \log \frac{w + |\Delta w_c|}{w}$, hence it follows that $\frac{w + |\Delta w_c|}{w} = 1.298$. This means that the mean residual ratio, after regression, for this case is $\frac{|\Delta w_c|}{w} \cong 30\%$. Analysis of the actual residuals for the soundings shown indicates that the majority of this error-ratio occurs at the highest altitudes recorded in the sounding where w is the smallest and where actual instrument accuracy is doubtful.

TABLE III

Values of λ and related statistics
for the data-sample for Waycross, Ga.

DATE DAY-GMT	λ-VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R^2_{12}	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\|X}$
16-0000	2.9414	0.2585	.953	.908	0.5861	0.1837
16-1200*	1.5298	0.4277	.704	.496	0.4130	0.3043
17-0000	4.5798	0.1950	.988	.976	0.8809	0.1387
17-1200	2.7855	0.1246	.987	.974	0.5373	0.0888
18-0000	2.2575	0.1187	.983	.966	0.4377	0.0846
18-1200	2.6057	0.1371	.983	.966	0.5057	0.0978
19-0000	2.3259	0.3495	.879	.773	0.5038	0.2491
19-1200	1.8176	0.2879	.868	.755	0.3981	0.2049
20-0000	3.6279	0.2036	.980	.960	0.7030	0.1447
20-1200	2.5391	0.2308	.950	.903	0.5083	0.1643
21-0000	3.0887	0.1589	.983	.966	0.5976	0.1131
21-1200	2.4250	0.1087	.987	.974	0.4680	0.0775
22-0000*	0.9815	0.3252	.642	.412	0.2911	0.2316
22-1200	3.2945	0.2850	.955	.912	0.6572	0.2031
23-0000	4.3912	0.2107	.985	.970	0.8473	0.1499
23-1200	4.5008	0.2425	.982	.964	0.8713	0.1724
TIME MEAN	2.8557			.869		
STD. DEV. $\sigma_t(\lambda)$	1.0401					

*Corresponds to $R_{12} < .85$.

TABLE IV

Values of λ and related statistics for the data-sample for Charleston, S. C.

DATE DAY-GMT	λ-VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σY	STANDARD ERROR $S_{Y\|X}$
16-0000	2.3199	0.1174	.984	.968	0.4485	0.0863
16-1200	4.33721	0.2933	.972	.945	0.8494	0.2088
17-0000	3.9920	0.1322	.993	.986	0.7646	0.0941
17-1200	3.8305	0.1629	.989	.978	0.7379	0.1160
18-0000	2.4849	0.1724	.970	.941	0.4883	0.1229
18-1200	2.7495	0.1236	.987	.974	0.5319	0.0883
19-0000	2.9405	0.1308	.988	.976	0.5684	0.0934
19-1200*	1.5187	0.4050	.721	.520	0.4013	0.2886
20-0000	4.5054	0.2364	.983	.966	0.8711	0.1680
20-1200	2.7135	0.2153	.961	.924	0.5369	0.1532
21-0000	4.7245	0.1834	.990	.980	0.9077	0.1305
21-1200	2.4194	0.1502	.976	.953	0.4727	0.1071
22-0000	MISSING DATA					
22-1200	2.5801	0.2807	.931	.867	0.5281	0.2001
23-0000	4.4411	0.2290	.983	.966	0.8597	0.1631
23-1200	3.5360	0.1560	.988	.976	0.6809	0.1110
TIME MEAN	3.2729			.928		
STD. DEV. $\sigma_t(\lambda)$	0.9832					

*Corresponds to $R_{12} < .85$.

TABLE V

Values of λ and related statistics for the data-sample for Lake Charles, La.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\|X}$
16-0000	2.9224	0.1461	.984	.968	0.5653	0.1041
16-1200	4.0954	0.1065	.996	.992	0.7837	0.0759
17-0000	4.5898	0.1810	.990	.980	0.8835	0.1291
17-1200	3.5190	0.2167	.976	.953	0.6878	0.1547
18-0000	4.0538	0.1916	.986	.972	0.7843	0.1367
18-1200	3.8884	0.1964	.984	.968	0.7528	0.1399
19-0000	2.7501	0.0960	.992	.984	0.5273	0.0684
19-1200	3.0506	0.2307	.965	.931	0.6030	0.1646
20-0000	2.4835	0.0959	.990	.980	0.4782	0.0684
20-1200	2.8351	0.1944	.971	.943	0.5575	0.1388
21-0000*	1.1751	0.2333	.813	.661	0.2757	0.1665
21-1200	1.6059	0.2349	.885	.783	0.3462	0.1676
22-0000	4.3386	0.1766	.992	.984	0.7777	0.1041
22-1200	5.2771	0.1789	.994	.988	0.8705	0.1016
23-0000	4.4083	0.2264	.983	.966	0.8532	0.1612
23-1200	4.5449	0.3695	.963	.927	0.8417	0.2375
TIME MEAN	3.4711			.936		
STD. DEV. $\sigma_t(\lambda)$	1.1352					

*Corresponds to $R_{12} < .85$.

TABLE VI

Values of λ and related statistics
for the data-sample for El Paso, Texas.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\|X}$
16-0000	3.5147	0.3078	.964	.929	0.5391	0.1509
16-1200	3.3290	0.2334	.976	.953	0.5050	0.1147
17-0000	3.4696	0.1695	.988	.976	0.5195	0.0832
17-1200	3.2455	0.3847	.936	.876	0.5123	0.1886
18-0000	3.7536	0.2578	.997	.994	0.5658	0.1260
18-1200	3.3012	0.1721	.987	.970	0.4938	0.0843
19-0000	3.3597	0.1046	.995	.990	0.4990	0.0513
19-1200	2.4246	0.2875	.936	.876	0.3841	0.1414
20-0000	2.5997	0.2698	.950	.902	0.4057	0.1327
20-1200*	0.6696	0.1802	.778	.605	0.1311	0.0868
21-0000	2.6959	0.3116	.939	.882	0.4232	0.1524
21-1200	3.1878	0.1860	.983	.966	0.4787	0.0911
22-0000	3.3213	0.2029	.982	.964	0.4995	0.0994
22-1200	2.4686	0.3511	.912	.832	0.4003	0.1722
23-0000	2.3344	0.1755	.973	.947	0.3545	0.0860
23-1200*	0.7583	0.4089	.526	.277	0.2192	0.1965
TIME MEAN	2.7771			.871		
STD. DEV. $\sigma_t(\lambda)$	0.9147					

*Corresponds to $R_{12} < .85$.

TABLE VII

Values of λ and related statistics for the data-sample for San Diego, Calif.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\|X}$
16-0000	3.5187	0.0897	.996	.992	0.6713	0.0637
16-1200	3.8860	0.1442	.991	.982	0.7444	0.1024
17-0000	3.4676	0.0874	.996	.992	0.6606	0.0620
17-1200	3.2899	0.1467	.988	.976	0.5932	0.0943
18-0000	3.9886	0.1914	.987	.974	0.7205	0.1230
18-1200	4.0171	0.1426	.992	.984	0.7684	0.1012
19-0000	3.6748	0.1190	.993	.986	0.7023	0.0845
19-1200	2.4739	0.1570	.975	.951	0.4817	0.1115
20-0000*	1.9672	0.3994	.807	.651	0.4627	0.2836
20-1200	3.9124	0.1845	.986	.972	0.7529	0.1309
21-0000	3.3273	0.2063	.976	.953	0.6470	0.1465
21-1200	3.8186	0.1946	.984	.968	0.7370	0.1382
22-0000	2.6873	0.2288	.956	.914	0.5338	0.1626
22-1200	2.3642	0.2423	.938	.880	0.4786	0.1721
23-0000	2.1203	0.2521	.919	.845	0.4378	0.1790
23-1200	2.8397	0.1020	.992	.984	0.5434	0.0725
TIME MEAN	3.2096			.938		
STD. DEV. $\sigma_t(\lambda)$	0.7012					

*Corresponds to $R_{12} < .85$.

TABLE VIII

Values of λ and related statistics for the data-sample for Caribou, Maine.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000*	1.3979	0.2714	.841	.707	0.2937	0.1661
16-1200	4.3499	0.0839	.998	.996	0.7705	0.0534
17-0000	3.5403	0.1644	.988	.976	0.6588	0.1047
17-1200	2.9646	0.1566	.984	.968	0.5350	0.1003
18-0000	3.1736	0.2673	.960	.922	0.5869	0.1711
18-1200*	1.5775	0.4351	.723	.523	0.3878	0.2788
19-0000	4.6004	0.1396	.995	.990	0.8453	0.0884
19-1200	3.7880	0.2120	.982	.964	0.6859	0.1359
20-0000*	1.0878	0.3522	.666	.444	0.2901	0.2254
20-1200	2.7297	0.2756	.957	.916	0.4660	0.1424
21-0000	5.0908	0.3520	.979	.958	0.9154	0.1960
21-1200	5.1735	0.2328	.989	.978	0.9479	0.1461
22-0000	3.7348	0.0533	.999	.998	0.6789	0.0335
22-1200	4.5463	0.1532	.994	.988	0.8320	0.0965
23-0000	6.5325	0.5259	.975	.951	1.1656	0.2745
23-1200	4.5879	0.1398	.996	.992	0.8014	0.0769
TIME MEAN	3.6797			.892		
STD. DEV. $\sigma_t(\lambda)$	1.4896					

*Corresponds to $R_{12} < .85$.

TABLE IX

Values of λ and related statistics for the data-sample for Maniwaki, Que.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	5.5686	0.3228	.981	.962	1.0019	0.2053
16-1200	3.0656	0.2679	.971	.943	0.4894	0.1246
17-0000	3.6455	0.1713	.987	.974	0.6561	0.1097
17-1200	3.2185	0.2048	.977	.955	0.5867	0.1315
18-0000	3.2674	0.2213	.976	.953	0.6138	0.1405
18-1200	2.8017	0.1775	.977	.955	0.5597	0.1249
19-0000	4.2382	0.1084	.996	.992	0.8075	0.0770
19-1200*	1.1753	0.3235	.724	.524	0.2892	0.2077
20-0000	1.8514	0.4214	.874	.764	0.3945	0.2075
20-1200	3.0911	0.1692	.987	.974	0.5533	0.0945
21-0000	3.2247	0.0641	.999	.998	0.5703	0.0320
21-1200	2.8132	0.1215	.991	.982	0.5029	0.0714
22-0000	2.3573	0.2628	.933	.870	0.4481	0.1680
22-1200	2.6480	0.1180	.989	.978	0.4888	0.0747
23-0000	3.4791	0.1644	.989	.978	0.6695	0.1038
23-1200	3.8057	0.3301	.971	.943	0.6607	0.1670
TIME MEAN	3.1407			.922		
STD. DEV. $\sigma_t(\lambda)$	0.9849					

*Corresponds to $R_{12} < .85$.

TABLE X

Values of λ and related statistics
for the data-sample for Sault-Ste-Marie, Mich.

DATE DAY-GMT	λ-VALUE CALCULATED	STANDARD ERROR OF λ-VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	2.4004	0.3536	.932	.869	0.4486	0.1741
16-1200	2.4229	0.1205	.986	.972	0.4355	0.0770
17-0000	4.3460	0.1218	.995	.990	0.7756	0.0780
17-1200	3.6100	0.1099	.995	.990	0.6639	0.0697
18-0000	3.7061	0.1273	.994	.988	0.6828	0.0807
18-1200	3.5366	0.1136	.994	.988	0.6515	0.0721
19-0000	2.0358	0.2896	.904	.817	0.3889	0.1733
19-1200	2.2482	0.2289	.943	.889	0.4225	0.1463
20-0000	2.8100	0.1758	.979	.958	0.5218	0.1108
20-1200	3.2512	0.0385	.999	.998	0.5917	0.0242
21-0000	3.3835	0.1394	.991	.982	0.6221	0.0879
21-1200	2.1833	0.2288	.945	.893	0.4217	0.1446
22-0000	3.0157	0.1823	.981	.962	0.5614	0.1153
22-1200	3.3318	0.1228	.993	.986	0.6384	0.0775
23-0000	3.8431	0.1015	.997	.994	0.7333	0.0640
23-1200	3.7412	0.1077	.996	.992	0.7496	0.0680
TIME MEAN	3.1166			.954		
STD. DEV. $\sigma_t(\lambda)$	0.6940					

TABLE XI

Values of λ and related statistics for the data-sample for Bismarck, N. D.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	2.6549	0.2499	.955	.912	0.5027	0.1564
16-1200	3.4759	0.2024	.984	.968	0.6213	0.1180
17-0000	6.5097	0.1724	.996	.992	1.1819	0.1080
17-1200	NO DATA AVAILABLE					
18-0000	NO DATA AVAILABLE					
18-1200	3.6856	0.2127	.984	.968	0.6601	0.1243
19-0000	4.4516	0.2273	.986	.972	0.8179	0.1427
19-1200	4.5117	0.1570	.995	.990	0.8724	0.0955
20-0000	3.4355	0.1859	.983	.966	0.6154	0.1180
20-1200	2.0076	0.2615	.918	.843	0.3888	0.1611
21-0000	4.9089	0.1923	.992	.984	0.8086	0.1088
21-1200	4.5735	0.1341	.996	.992	0.7678	0.0743
22-0000	5.0893	0.2730	.987	.974	0.9794	0.1640
22-1200	3.8804	0.0609	.999	.998	0.7341	0.0382
23-0000	3.6326	0.3285	.954	.910	0.6710	0.2088
23-1200	2.1195	0.1005	.987	.974	0.3789	0.0639
TIME MEAN	3.9240			.960		
STD. DEV.: $\sigma_t(\lambda)$	1.2147					

TABLE XII

Values of λ and related statistics for the data-sample for Glasgow, Mont.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	3.7842	0.1697	.990	.980	0.6406	0.0943
16-1200	6.6178	0.3483	.986	.972	1.1253	0.1938
17-0000	5.6933	0.1384	.997	.994	0.9578	0.0770
17-1200	6.0083	0.2198	.993	.986	1.0154	0.1224
18-0000	6.0189	0.1978	.995	.990	1.0630	0.1100
18-1200	5.1013	0.1940	.993	.986	0.8650	0.1083
19-0000	6.0881	0.1843	.995	.990	1.0284	0.1028
19-1200	2.6928	0.2074	.972	.945	0.4666	0.1158
20-0000	3.3309	0.2818	.963	.927	0.5644	0.1593
20-1200	2.2316	0.1834	.965	.931	0.3754	0.1031
21-0000	5.3035	0.2107	.991	.982	0.8700	0.1187
21-1200	3.1120	0.3134	.953	.908	0.5487	0.1746
22-0000	4.4330	0.1457	.995	.990	0.7500	0.0813
22-1200	3.3728	0.1415	.991	.982	0.5573	0.0802
23-0000*	0.6804	0.3303	.614	.377	0.1954	0.1649
23-1200*	0.8355	0.2358	.746	.557	0.1885	0.1316
TIME MEAN	4.0815			.906		
STD. DEV. $\sigma_t(\lambda)$	1.8749					

*Corresponds to $R_{12} < .85$.

TABLE XIII

Values of λ and related statistics
for the data-sample for Spokane, Wash.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	6.1242	0.2171	.994	.988	1.0350	0.1209
16-1200	5.3894	0.0292	.999	.998	0.9060	0.0163
17-0000	6.2015	0.2579	.992	.984	1.0971	0.1432
17-1200	5.4573	0.0661	.999	.998	0.9586	0.0367
18-0000	4.1476	0.2953	.973	.947	0.6969	0.1673
18-1200	3.1086	0.2754	.959	.920	0.5305	0.1562
19-0000	3.3134	0.1454	.990	.980	0.5484	0.0825
19-1200	2.5267	0.1350	.985	.970	0.4199	0.0766
20-0000	2.6138	0.1618	.980	.960	0.4352	0.0914
20-1200	4.5322	0.4601	.948	.899	0.7780	0.2593
21-0000	4.2112	0.3912	.956	.914	0.7166	0.2204
21-1200	3.6604	0.2030	.984	.968	0.6057	0.1145
22-0000*	1.6059	0.3356	.822	.676	0.3184	0.1894
22-1200*	1.0266	0.2085	.829	.687	0.2015	0.1176
23-0000	5.3603	0.2307	.990	.980	0.8799	0.1299
23-1200	4.0725	0.4338	.968	.937	0.5157	0.1406
TIME MEAN	3.9594			.925		
STD. DEV. $\sigma_t(\lambda)$	1.5414					

*Corresponds to $R_{12} < .85$.

TABLE XIV

Values of λ and related statistics
for the data-sample for Quillayute, Wash.

DATE DAY-GMT	λ -VALUE CALCULATED	STANDARD ERROR OF λ -VALUE	CORRELATION COEFFICIENT R_{12}	R_{12}^2	STANDARD DEVIATION σ_Y	STANDARD ERROR $S_{Y\mid X}$
16-0000	5.5314	0.1401	.996	.992	1.0899	0.0992
16-1200	5.5505	0.1507	.996	.992	1.1368	0.1065
17-0000	4.3589	0.2506	.979	.958	0.8494	0.1789
17-1200*	1.4652	0.3352	.771	.594	0.3626	0.2394
18-0000*	1.7747	0.3777	.793	.629	0.4275	0.2701
18-1200	1.6247	0.2586	.867	.752	0.3580	0.1850
19-0000	2.0580	0.1481	.968	.937	0.4061	0.1058
19-1200	2.3200	0.1796	.963	.927	0.4596	0.1282
20-0000	2.5319	0.1793	.969	.939	0.4977	0.1278
20-1200	3.6523	0.3188	.954	.910	0.7291	0.2272
21-0000*	1.6818	0.2995	.842	.709	0.3805	0.2133
21-1200	1.4532	0.3086	.794	.630	0.3485	0.2199
22-0000	1.7184	0.2594	.878	.771	0.3722	0.1846
22-1200	2.0152	0.1758	.982	.964	0.4858	0.1019
23-0000	4.1114	0.1946	.986	.972	0.7917	0.1382
23-1200	3.9581	0.3402	.972	.945	0.7002	0.1755
TIME MEAN	2.8629			.851		
STD. DEV. $\sigma_t(\lambda)$	1.5414					

*Corresponds to R_{12} < .85.

In table III, a low correlation of 0.642 occurs at 22 March, 0000GMT, which results in a relatively high standard error $S_{Y|X}$. Note in Table III that most of the profiles have $R_{12} > 0.95$ with associated small standard errors. There were two cases in Table III with low correlation coefficients, specifically 0.704 and 0.642. This pattern of correlation coefficients $R_{12} \geq 0.85$ except for a possible set of one to three profiles for which R_{12} fell below this value, continues throughout all of the 12 stations considered here as shown in Tables III through XIV. Specific instances in these tables were the R_{12}-value value drops below 0.85 have been marked by an asterisk alongside of the date. In all of these cases the parameter λ is likewise smaller than normal in view of Eq. (20)

$$\lambda = R_{12}(\sigma_Y / \sigma_X) \tag{44}$$

All of these special cases where $R_{12} \leq 0.85$ are characterized by a <u>standard deviation</u> σ_Y <u>lower than</u> typical for the station. Such cases of small σ_Y-values represented cases where statistically there was insufficient vertical variation in the moisture range to ascribe it successfully to the power-profile parameterization.

An example of a smaller-than-normal correlation coefficient R_{12} and its correspondingly small λ has been extracted from the regression print out in the form of Table XV. This table lists the statistical results from Waycross, Ga., at 0000GMT, 22 March, 1971. This power-profile analysis resulted in $R_{12} = 0.642$ and $\lambda = 0.9815$, which was considerably lower than the mean λ-value. In Table XV, the observed moisture profile is indicated in column 2 as Y of Eq. (7), and in column 3 is shown the corresponding $\hat{Y}$ of Eq. (40). The residual shown in column 4 results from forming $Y-\hat{Y}$ from Eq. (41). As expected, $\hat{Y}$ began as a maximum positive value at the surface and decreased to 0.000 at 500 mb and becomes negative

TABLE XV

An example for Waycross, Ga. (0000 GMT, 22 Mar., 1971) showing a condition leading to an ill-defined λ -profile.

PRESSURE (mb)	Y (obs)	$\hat{Y}$	RESIDUAL
1014	0.4660	0.3014	0.1646
1000	0.3179	0.2955	0.0224
950	0.2719	0.2736	-0.0017
900	0.2478	0.2505	-0.0028
850	0.2020	0.2262	-0.0241
800	0.1024	0.2003	-0.0979
750	-0.0060	0.1728	-0.1789
700	-0.1507	0.1434	-0.2942
650	-0.1579	0.1118	-0.2716
600	0.1296	0.0777	0.0519
550	-0.0082	0.0406	-0.0488
500	0.0000	0.0000	0.0000
450	-0.3650	-0.0449	-0.3201
400	-0.6949	-0.0951	-0.5998

TABLE XVI

Environmental parameters used in the calculation of Table XV.

PRESSURE (mb)	TEMP (C)	RELATIVE HUMIDITY, %	MIXING RATIO (g/kg)
1014	16.8	33	3.8801
1000	15.6	25	2.7590
950	12.0	27	2.4816
900	8.3	31	2.3477
850	6.9	29	2.1130
800	4.3	26	1.6800
750	1.6	23	1.3087
700	- .5	18	.9378
650	- 2.9	20	.9186
600	- 6.3	48	1.7885
550	-10.9	48	1.3021
500	-16.9	77	1.3270
450	-19.7	39	.5726
400	-27.2	34	.2678

above 500 mb. The observed profile, $Y(p)$, began also as a positive value at the surface, but becomes negative in the layer 750 to 600 mbs indicating that the actual mixing ratio profile had values less than w_{500} in that layer.

Table XVI lists the actual atmospheric-sounding parameters which resulted in the values shown in Table XV. Note that the reported relative humidities are low in the 750 to 650 mb layer and high in the layer 600 to 450 mb. Since by Eq. (9), w is a function of relative humidity as well as of temperature and pressure, relatively low w-values exist from 750 to 650 mb and high w-values in the 600- to 450 mb- range. This, at least locally, is inconsistent with the assumption on which the lower-fit is based and, as calculated, gives a low value of R_{12}. Note that the power-fit need not have been inconsistent if the temperature lapse was large enough through the 600 - 450 mb layer, which in this instance was not the case.

As just noted, the second major factor which controls the consistency of the profile-fit at level p is the temperature $T(p)$. It can also be shown, by example, that a temperature inversion or an isothermal layer may be (depending upon relative humidity) associated with w values which do not decrease in height, resulting in the low correlation coefficients R_{12}. In Tables III to XIV, all cases where $R_{12} \geq 0.85$ which thus led to an ill-defined (i.e., too small) value of λ are denoted by an asterisk. The explanation of all of these special cases is similar to that advanced for the Waycross, Ga. case in Tables XV, XVI.

Note finally that it is convenient to list the time-mean $\bar{\lambda}$ and R_{12}^2 for each station listed in Tables III,..., XIV. In addition it is desirable to compute the temporal standard deviation of the λ's in each table according to

$$\sigma_t(\lambda) = \left[\frac{(\lambda_1 - \bar{\lambda})^2 + \ldots + (\lambda_{16} - \bar{\lambda})^2}{15}\right]^{\frac{1}{2}} \tag{45}$$

B. REDUCTION OF VARIANCE OF λ

Having obtained the sixteen λ -values per station the next portion of the investigation involved determining the feasibility of reducing the variance of λ for each station over the period of time considered using other sounding parameters. The synoptic variables introduced into the computation as independent variables permitted utilization of the stepwise multiple regression aspect of the BIMED 02R program where λ was the dependent variable. In addition to the 16 values of λ per station, 16 values of each of the following X_i were read into the program as independent variables:

surface temperature defined as X_1

surface mixing ratio defined as X_2

850 mb temperature defined as X_3

850 mb mixing ratio defined as X_4

700 mb temperature defined as X_5

500 mb temperature defined as X_6.

The above variables were used in the multiple regression as inputs with the computed λ as the dependent variable in each case. The desired solution equation would then be of the form of Eq. (32) where $Y_1 = \lambda$, and the X_i are the respective independent variables. The stepwise regression program BIMED 02R was then utilized to derive the best-fit form of

$$\lambda = A_0 + A_1X_1 + A_2X_2 + \ldots + A_6X_6 \tag{46}$$

Among the factors which might be considered as useful in reducing the temporal variance of λ are variations in the synoptic patterns, and those which may reflect purely local conditions. Among the latter effects

(e.g., Tables III,..., XIV), station location and elevation should be considered, as well as proximity to large water bodies and lakes. In the sounding variables $X_1,...,X_6$, the lower-level parameters, were considered to be primarily representative of local effects, whereas the higher-level variables were presumed to be indicative of the larger-scale synoptic time-scales, although there is undoubtedly some interaction between the low- and high-level variables.

The temporal standard deviations $\sigma_t(\lambda)$ listed at the base of Tables III,...,XIV resulting from Eq. (45) ranged from minimal values of $\sigma_t(\lambda) = 0.694$ at Sault Ste Marie, and $\sigma_t(\lambda) = 0.701$ at San Diego to a maximum $\sigma_t(\lambda) = 1.875$ at Glasgow, Montana. In general it can be seen that stations close to large bodies of water (Charleston, San Diego, Maniwaki, and Sault Ste Marie) have relatively small standard deviations whereas stations away from bodies of water have larger standard deviations (Bismarck, Glasgow, and Spokane). The cases of large values of $\sigma_t(\lambda)$ seem to afford considerable opportunity for utilization of the regression procedure for the reduction of the λ-variance. However, it was decided to test the diagnostic usefulness of Eq. (46) for all twelve stations, regardless of the $\sigma_t(\lambda)$ value.

A preliminary test run was conducted using the BIMED 02R program with the variables defined previously in the section. During this run all independent variables were allowed to enter freely based solely on their partial correlation values. The program was allowed to run until the requirement of 2% added fractional explained variance after introduction of the kth variable could not be met.

Two facts became evident from this preliminary test: (1) the independent variable with the highest initial correlation coefficient in almost

all cases was X_4, and (2) the maximum number of significant steps necessary to ensure the required minimum added explained variance at each step was four. At steps five and six of the preliminary run the added explained variance upon predictor-entry was too small. Since a low-level moisture predictor was desired to be compatible with the variations of the λ - profile parameter (cf., the discussion for Waycross at 0000 GMT, 22 March, 1971), it was decided to utilize the control-delete option of BIMED 02R to force in X_4 as the first predictor at each station. The second option used was that of limiting the number of predictors to four in each regression analysis. The results of the sets of regressions over the 12 stations are shown in Table XVII.

Equation (42) gives the relationship between the standard error of λ , after the four-step multiple correlation R_m has been determined

$$S_{\lambda|X} = (1 - R_m^2)^{\frac{1}{2}} \quad \sigma_t(\lambda) \tag{47}$$

From Table XVII it can be seen that when $\sigma_t(\lambda)$ is relatively small, the explained fraction of this statistic is relatively small. The result, then, of application of the multiple regression technique is to reduce the standard deviation $\sigma_t(\lambda)$ of all stations, but to reduce the stations with the largest $\sigma_t(\lambda)$ the most.

San Diego is a good example of a station having a small $\sigma_t(\lambda)$, $\sigma_t(\lambda) = 0.7012$, which means, according to the above result, a relatively small reduction, specifically to $S_{\lambda|X}=0.5686$. Correspondingly, this small additional reduction is reflected in the relatively low multiple correlation coefficient at San Diego of $R_m = .5852$. At the other extreme is Glasgow having a temporal standard deviation $\sigma_t(\lambda) = 1.8749$. By the multiple regression procedure a standard error of estimate of 0.8745 results. Again this is consistent with the high multiple correlation coefficient, $R_m = 0.8846$.

TABLE XVII

Stepwise regression parameters defining λ in terms of sounding parameters for the data period (16 soundings).

STATION	MULT. CORR. COEF. R_m	FRACT. EXPL. VAR. R_m^2	STANDARD ERROR $S_{\lambda\|X}$	STANDARD DEVIATION $\sigma_t(\lambda)$	PREDICTOR ENTRY X_i
Waycross	.7214	.5204	.7203	1.040	4,6,2,5
Charleston	.7044	.4962	.6979	0.9832	4,6,1,2
Lake Charles	.8221	.6758	.6464	1.135	4,2,6,5
El Paso	.8211	.6742	.5222	0.9147	4,5,6,2
San Diego	.5852	.3425	.5686	0.7012	4,1,2,6
Caribou	.7890	.6225	.9152	1.4896	4,5,2,3
Maniwaki	.6256	.3914	.7684	0.9850	4,3,1,2
Sault Ste Marie	.6710	.4502	.5146	0.6940	4,6,2,3
Bismarck	.9086	.8256	.5073	1.2147	4,6,1,3
Glasgow	.8846	.7825	.8745	1.8749	4,6,2,3
Spokane	.9205	.8473	.6024	1.5414	4,6,1,2
Quillayute	.9309	.8666	.5259	1.4398	4,6,2,3
MEAN	.7820	.6246	.6553	1.1394	

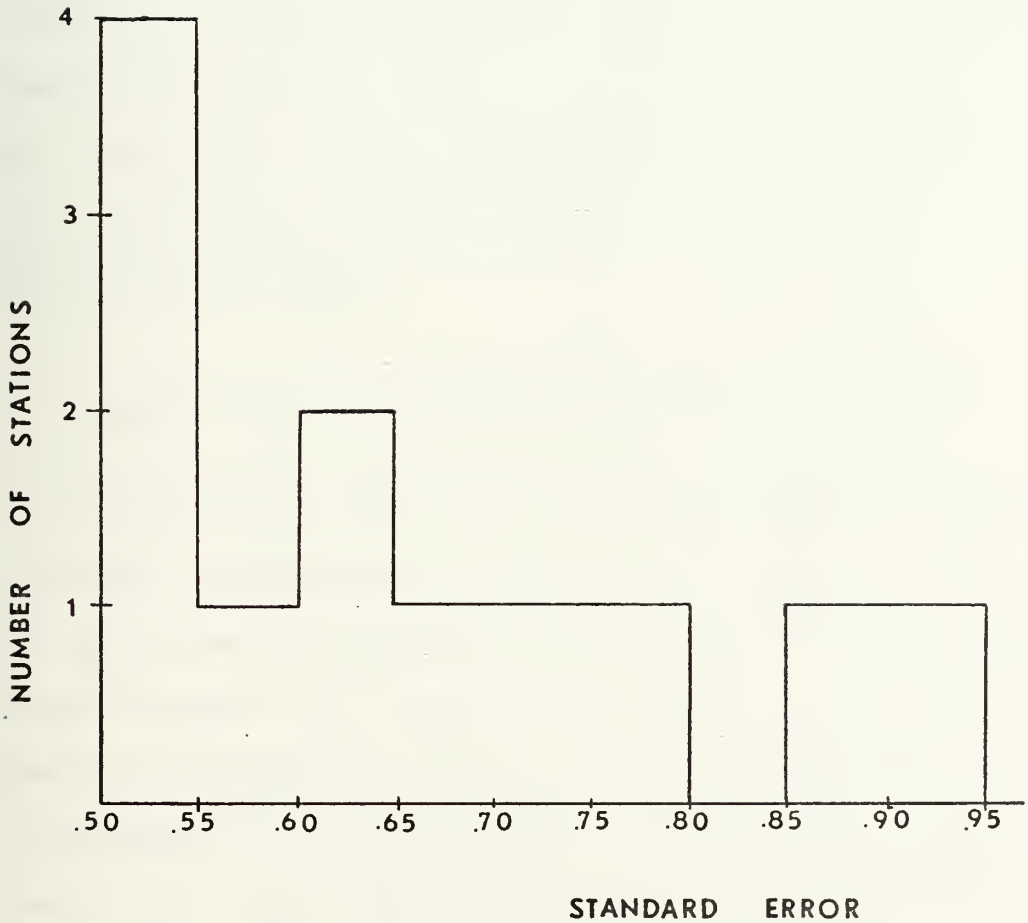

Figure 2: Histogram depicting number of stations within certain standard error limits.

The result of the multiple regression for each station is to give a reduced standard error of λ . A histogram depicting the 12 standard error values is shown in Figure 2, on a scale of $S_{\lambda|X}$ from 0.50 to 0.95. Note that most of the standard errors after regression fall into the range 0.5 to 0.65.

The righthand column in Table XVII lists the order of entry of the independent variables into the stepwise regression and is based on the value of the partial correlation of the respective variables as explained in Section III (B). From Table XVII it may be noted that in seven of the 12 stations the second independent variable added to the regression was X_6. Also in seven of the 12 stations the third independent variable added was X_2. This led to the idea of possible classification of stations according to a forced order of entry, an option of BIMED 02R, and will be discussed in the next sub-section.

C. STATION CLASSIFICATION ACCORDING TO THE PREDICTOR ENTRY

Table XVIII lists an attempt to classify stations by the relative priority of the independent variables to the program specified in the order X_4, X_6, X_2, and the remaining predictor in a 4-step selection left as free choice according to the selection criterion. As can be seen by comparing Tables XVII and XVIII there was no change in the multiple correlation coefficient at 10 of the 12 stations as a result of specifying the 4-6-2 input order. This is because in almost all cases the variables X_4, X_6, and X_2 were in the original run (Table XVII) even though not necessarily in this specific order. Table XVIII lists the "single free choice" variable under order of input as the last independent variable in the list for each station.

TABLE XVIII

Stepwise regression parameters defining λ in terms of sounding parameter in a specified order for the data period (16 soundings).

STATION	MULT. CORR. COEFF. R_m	FRACT. EXPL. VAR. R_m^2	STANDARD ERROR	PREDICTOR ENTRY X_i
Waycross	.7214	.5204	.7203	4,6,2,5
Charleston	.7044	.4962	.6979	4,6,2,1
Lake Charles	.8221	.6758	.6464	4,6,2,5
El Paso	.8211	.6742	.5222	4,6,2,5
San Diego	.5852	.3425	.5686	4,6,2,3
Caribou	.7739	.5989	.9434	4,6,2,5
Maniwaki	.6256	.3914	.7684	4,6,2,1
Sault Ste Marie	.6710	.4502	.5146	4,6,2,3
Bismarck	.8787	.7721	.5799	4,6,2,1
Glasgow	.8846	.7825	.8745	4,6,2,3
Spokane	.9205	.8473	.6024	4,6,2,1
Quillayute	.9309	.8666	.5259	4 6,2,3
MEAN	.7782	.6181	.6637	

MEAN $\sigma_t(\lambda) = 1.1394$

It may be noted that there are four stations with surface temperature as the last independent variable (Charleston, Maniwaki, Bismarck, and Spokane), four with 500 mb-temperature (Waycross, Lake Charles El Paso, and Caribou), and four with 850 mb-temperature (San Diego, Sault Ste Marie, Glasgow, And Quillayute). There might, then, be some way of classifying or typing stations on the basis of the free-choice predictor selected at step 4. However, with the small station set available it was not clear that this could be done.

Finally, it is to be emphasized that Table XVII indicates that the observed λ-values tend to adjust to (a) the synoptic-scale time variations in the sounding and to (b) the local-effect variations. The latter effect is harder to specify so that one achieves some limiting accuracy in the expected value resulting from the regression. The magnitude of this accuracy is suggested by the standard error which tends to show a cluster of values in the range 0.50 to 0.65 (Fig. 2).

V. PRECIPITABLE WATER VAPOR TESTS

As noted in the introduction, two verification techniques were to be used to evaluate the results of this investigation. The first of these, namely verification of the moisture profile, has been discussed in detail in Section IV from a statistical viewpoint. In this section, an analysis of the verification of the moisture-parameterization technique from the viewpoint of precipitable water vapor is presented.

The formulas used to calculate the precipitable water are shown as Eqs. (12) and (13). Equation 12 was utilized to obtain precipitable water based upon the moisture profile of the data for the given data sample. It is the precipitable water from the observed sounding, and hence will be referred to as observed precipitable water, W_o. Equation 13 is used to calculate the precipitable water utilizing the λ-values already obtained. As can be seen from Eq. (13), it is based on only three variables: 500-mb mixing-ratio, surface pressure, and λ. There are, however, two possible methods since λ-values were obtained by the profile-parameter method and, secondly, by the multiple regression technique. Precipitable water calculated using the profile parameter λ-values, one for each sounding, will be designated as W_{cs} and those relatively smooth values obtained by using the multiple regression λ-values denoted by W_{cm}.

For this phase of the investigation, four stations were chosen primarily on the basis of their varying geographical locations: Waycross, Ga., in the southeastern United States; San Diego in the southwest; Spokane, Wash., in the northwest; and Sault Ste Marie, Mich., in the northeast. Results

of the precipitable water calculations are listed in Tables XIX through XXII and are plotted in Figures 3 through 6.

As can be seen from Tables XIX,...,XXII or from Figures 3,...,6, the W_{cs}-value is in almost all cases within approximately 0.05 cms of the W_o value. Needless to say, with such good verification, the trend of W_{cs} is the same as the observed trend. It is obvious from the results, then, that the use of the power-profile technique can be expected to yield precipitable water values which are very representative of the W_o values, i.e. within 0.05 cms (cf., Tables XIX,...,XXII).

The values of W_{cm} are also presented in Tables XIX,...,XXII and in Figures 3,...,6. As can be seen errors do exist in the W_{cm} values, assuming W_o is the correct value. These errors can at times become large, i.e. as much as 100%. In general, however, the W_{cm} values which result from using λ -values obtained by multiple regression follow the time trend of W_o. Large errors in W_{cm} are then the exception rather than the rule, and as can be seen in Figures 3,...,6, occur at only one or two sounding times per station. The soundings which give the largest W_{cm} errors are, in fact, just those soundings which had the lower values of λ and of the correlation coefficients R_{12} in connection with the W_{cs} computations.

The 0000 GMT, 22 March, 1971 sounding at Waycross, Ga. will be used to illustrate the point of a large W_{cm} error. As can be seen in Table III, this sounding gave a low correlation coefficient value, 0.642, and a λ -value of 0.9815 where the time-mean over the 16 soundings for that station was 2.8557. Figure 3 shows that the use of the λ -value from the power-fit profile, λ = 0.9815, in Eq. (13) gives an accurate verification of W_o. On the other hand the value of λ resulting from the multiple

TABLE XIX

Table of precipitable water vapor values (W_o, W_{cs}, W_{cm}) for Waycross, Ga.

DAY DAY - GMT	OBSERVED W_o	PROFILE W_{cs}	MULT. REG. W_{cm}
16 - 0000	3.7190	3.8704	3.0169
16 - 1200	1.0227	1.1884	1.2003
17 - 0000	0.9360	0.8618	0.4918
17 - 1200	0.5325	0.5421	0.5940
18 - 0000	0.5678	0.5863	0.5923
18 - 1200	0.6549	0.6600	0.5514
19 - 0000	1.1639	1.1672	1.6403
19 - 1200	2.2758	2.2377	1.9609
20 - 0000	1.0744	0.9818	0.6177
20 - 1200	0.5083	0.5216	0.5058
21 - 0000	0.4897	0.4766	0.5948
21 - 1200	0.8447	0.8370	0.9129
22 - 0000	0.9807	1.1674	2.4002
22 - 1200	1.2914	1.1747	1.1335
23 - 0000	1.5845	1.3996	0.9415
23 - 1200	3.0389	2.8514	2.1848

TABLE XX

Table of precipitable water vapor values (W_o, W_{cs}, W_{cm}) for San Diego, Calif.

DAY DAY - GMT	OBSERVED W_o	PROFILE W_{cs}	MULT. REG. W_{cm}
16 - 0000	0.7488	0.7849	0.5459
16 - 1200	0.8116	0.8317	0.6233
17 - 0000	0.8222	0.8833	0.6292
17 - 1200	0.8586	0.8796	0.8950
18 - 0000	0.8783	0.8523	0.7305
18 - 1200	1.0573	1.0334	0.7619
19 - 0000	0.9586	1.0137	0.8904
19 - 1200	1.1637	1.2238	1.6504
20 - 0000	0.7991	0.9855	1.5071
20 - 1200	0.7310	0.8049	0.8771
21 - 0000	0.6700	0.7594	0.8141
21 - 1200	0.9416	0.9233	0.8065
22 - 0000	0.7205	0.8268	1.0792
22 - 1200	1.2398	1.3404	1.8782
23 - 0000	1.1738	1.3048	1.7369
23 - 1200	1.7495	1.7309	1.4738

TABLE XXI

Table of precipitable water vapor values (W_o, W_{cs}, W_{cm}) for Spokane, Wash.

DAY DAY - GMT	OBSERVED W_o	PROFILE W_{cs}	MULT. REG. W_{cm}
16 - 0000	0.3970	0.3932	0.2822
16 - 1200	0.3309	0.3313	0.2976
17 - 0000	0.3965	0.4002	0.4135
17 - 1200	0.3807	0.3731	0.6097
18 - 0000	0.3491	0.3174	0.2582
18 - 1200	0.4091	0.3976	0.3855
19 - 0000	0.4692	0.4820	0.4334
19 - 1200	0.5359	0.5436	0.5835
20 - 0000	0.5297	0.5337	0.6059
20 - 1200	0.6018	0.4964	0.3805
21 - 0000	0.4113	0.3489	0.3469
21 - 1200	0.3094	0.2940	0.1853
22 - 0000	0.3098	0.3036	0.4224
22 - 1200	0.7075	0.7759	0.9983
22 - 0000	0.7988	0.7595	0.6473
23 - 1200	0.9039	1.0635	1.3750

TABLE XXI

Table of precipitable water vapor values (W_o, W_{cs}, W_{cm}) for Sault Ste Marie, Mich.

DAY DAY - GMT	OBSERVED W_o	PROFILE W_{cs}	MULT. REG. W_{cm}
16 - 0000	1.1656	1.1476	1.0986
16 - 1200	0.3112	0.3207	0.3864
17 - 0000	0.3320	0.3293	0.1813
17 - 1200	0.3511	0.3401	0.3188
18 - 0000	0.3865	0.3693	0.3514
18 - 1200	0.4089	0.3887	0.2807
19 - 0000	0.5890	0.5605	0.7646
19 - 1200	0.7004	0.7405	0.8876
20 - 0000	0.7978	0.7349	0.6034
20 - 1200	0.5112	0.5162	0.4981
21 - 0000	0.5046	0.5150	0.4953
21 - 1200	0.2910	0.2889	0.4080
22 - 0000	0.3454	0.3433	0.4346
22 - 1200	0.2752	0.2701	0.2942
23 - 0000	0.3232	0.3234	0.3215
23 - 1200	0.2074	0.2032	0.1921

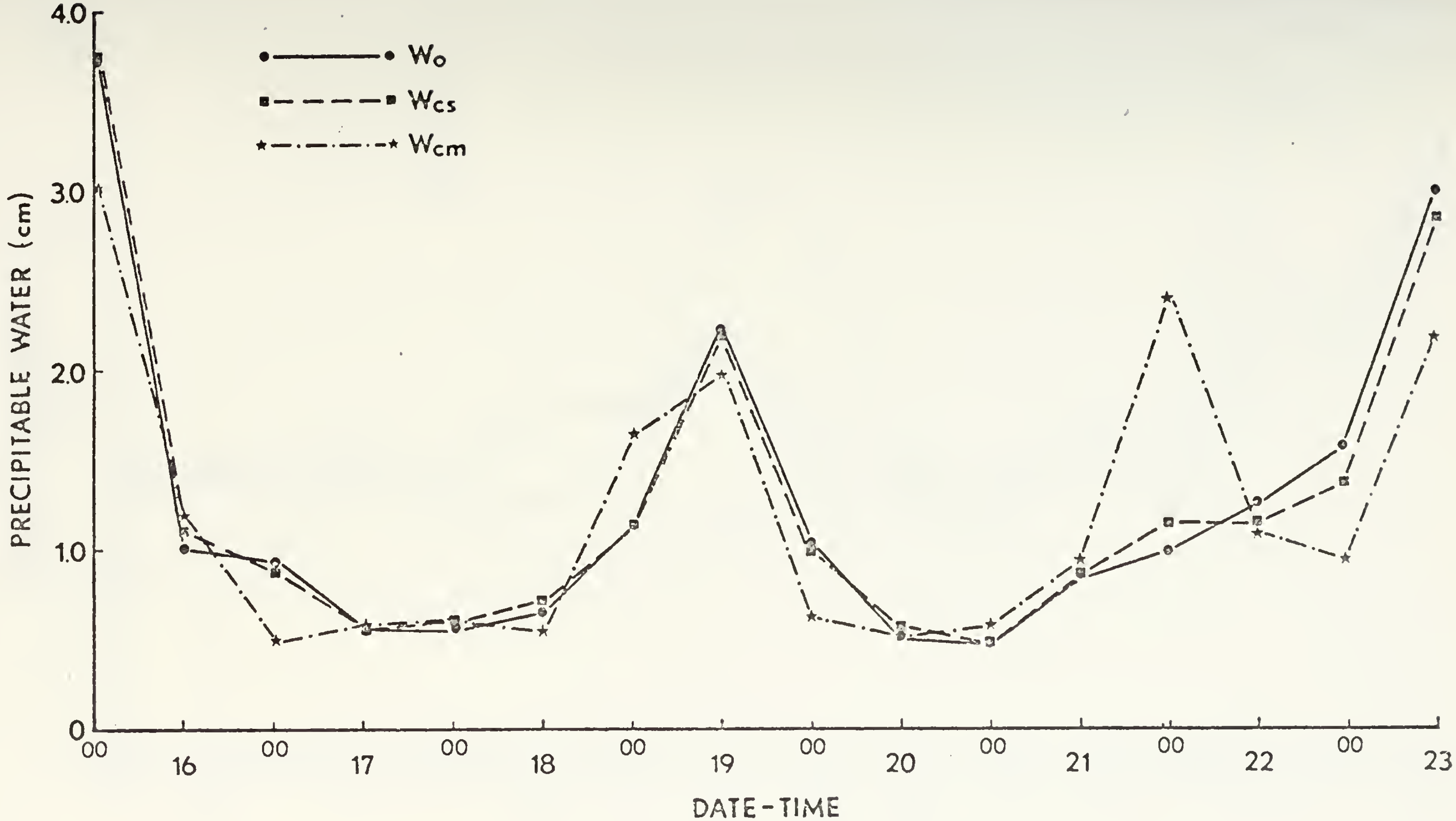

Figure 3: Plot of W_o values, W_{cs} values, and W_{cm} values over the period 16 - 23 March, 1971 for Waycross Ga.

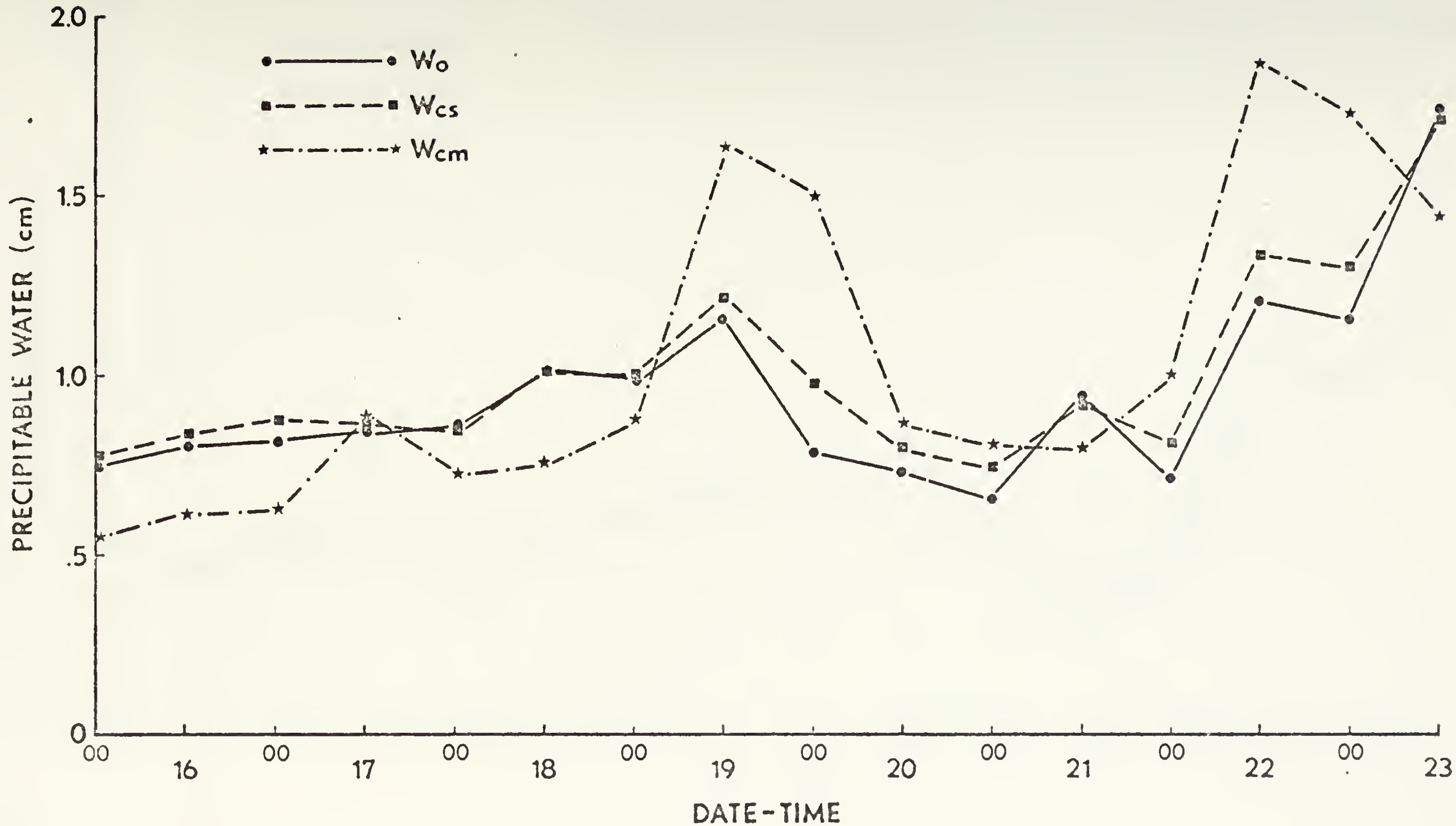

Figure 4: Plot of W_o values, W_{cs} values, and W_{cm} values over the period 16 - 23 March, 1971 for San Diego, Calif.

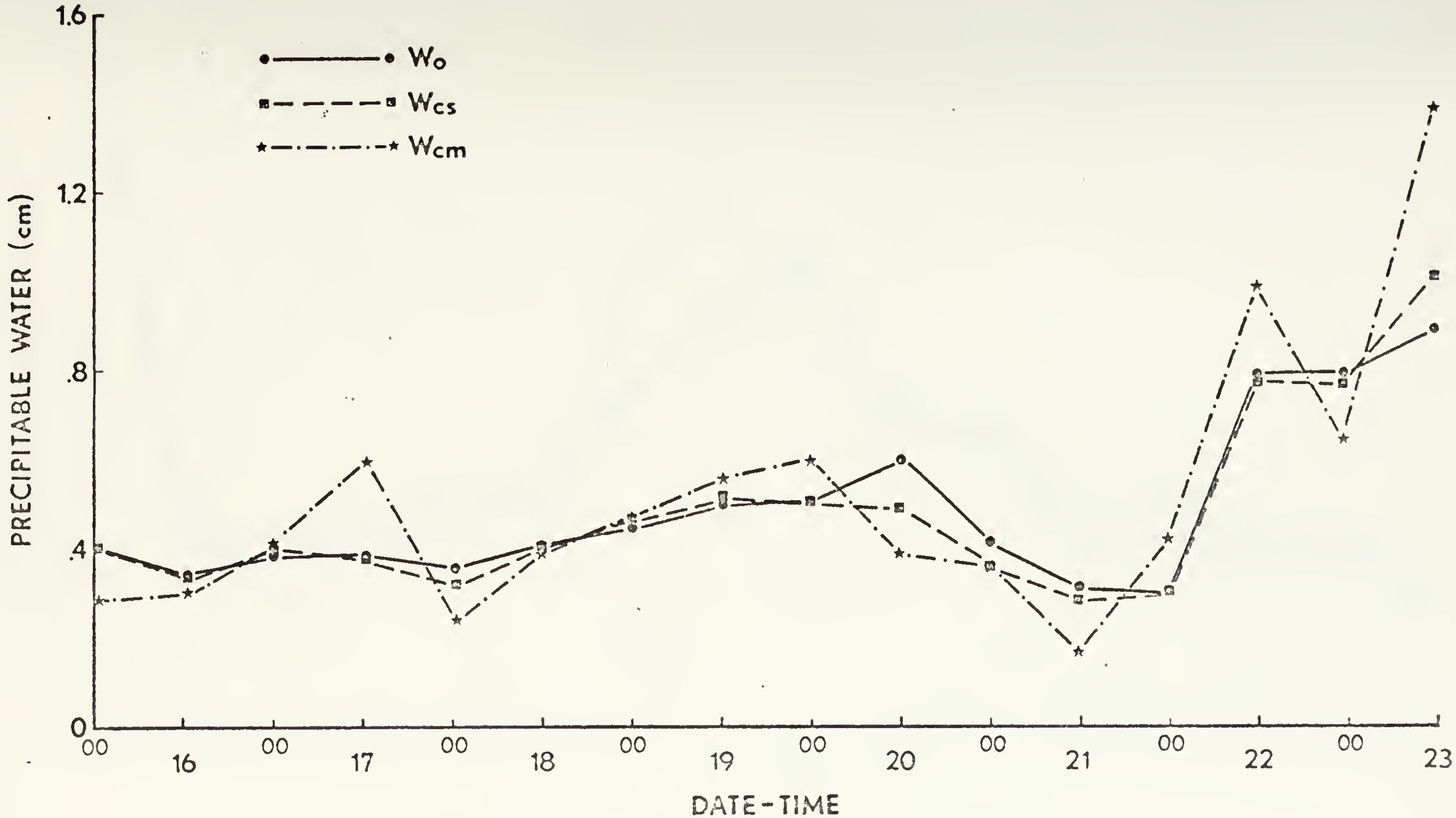

Figure 5: Plot of W_o values, W_{cs} values, and W_{cm} values over the period 16 - 23 March, 1971 for Spokane, Wash.

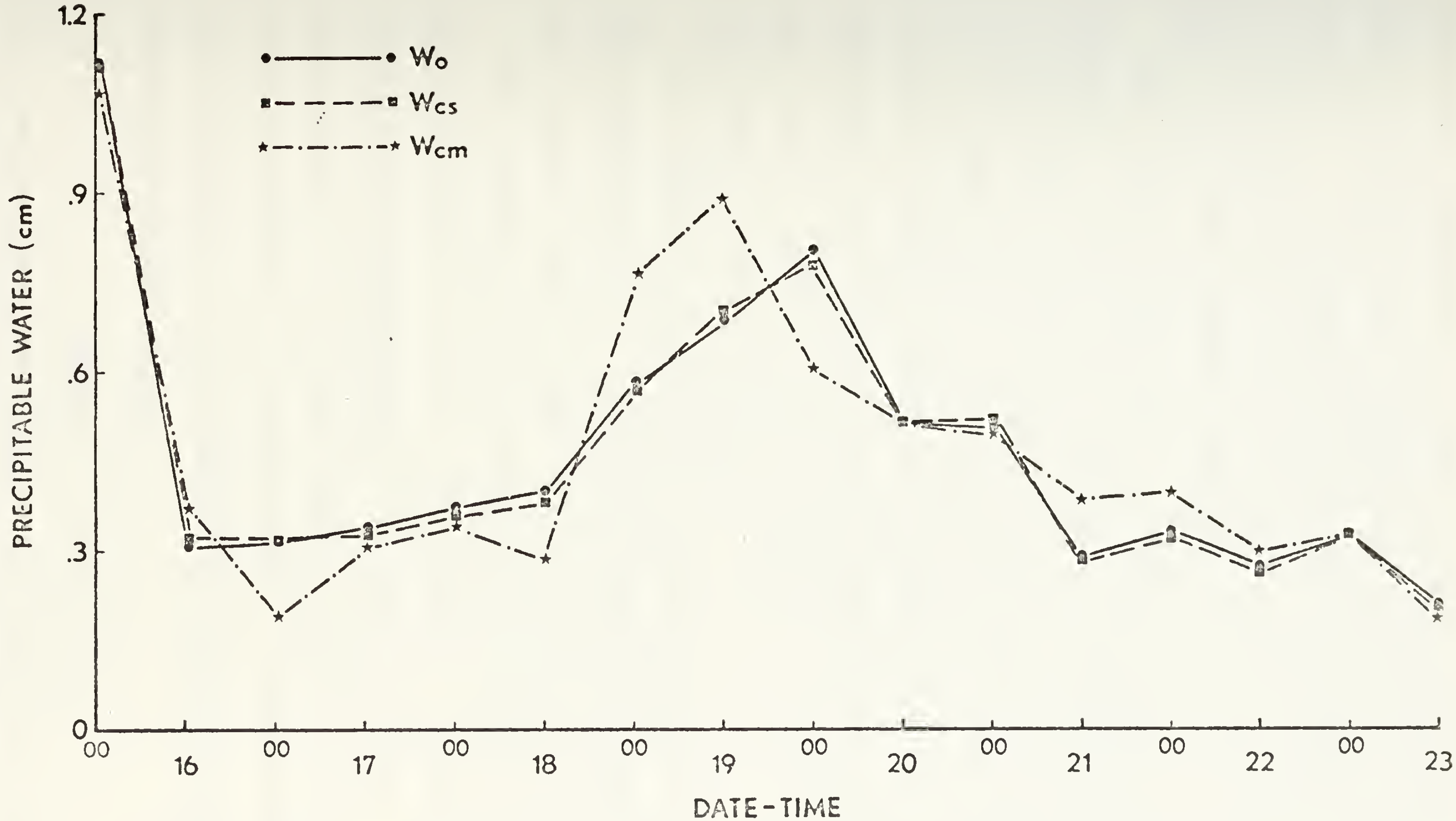

Figure 6: Plot of W_o values, W_{cs} values, and W_{cm} values over the period 16 - 23 March, 1971 for Sault Ste. Marie.

regression procedure for the date and station considered was $\lambda = 2.6793$. Since the latter value is considerably larger than the sounding best-fit value of λ, the application of the multiple regression λ-value in Eq. (13) gives a much larger value of W_{cm} than W_o; hence the large error, $W_{cm} - W_o$, at this sounding time (Figure 3).

The results of Tables XIX,...,XXII (and the four supporting diagrams Figures 3,...,6) have been presented in the statistically summarized form as Table XXIII. In Table XXIII, the mean W_o-values have been compared to the computed mean values of W_{cs} and W_{cm} using the 16 sounding samples at the four stations, Waycross, San Diego, Spokane, Sault Ste Marie. The results were by-products of the correlation of W_o separately against W_{cs}, and of W_o with W_{cm}, which was applied to the two sets of data in Tables XIX,...,XXII, respectively.

The standard deviations of W_o, W_{cs}, and W_{cm} have been summarized in Table XXIII (column 3). Note that the means and standard deviations of the W-samples computed by the different procedures are nearly equal.

The correlation coefficients $R(W_o, W_{cs})$ and $R(W_o, W_{cm})$ are listed for the four stations in column 4 of Table XXIII. The corresponding <u>coefficients of determination</u> R^2 are listed in column 5, and are to be interpreted as the fractional explained variance (of W_o). Finally the column "Std. Error" is to be interpreted as the mean dispersion of the curve of W_o minus W_{cs}, and also that of W_o minus W_{cm}, using the following equation for standard error:

$$S_{W_o|W_c} = (1 - R_c^2)^{\frac{1}{2}} \sigma_{W_o} \tag{48}$$

Note that the estimation of W_{cs} to give W_o leads to a standard error (a mean residual) of 0.07 cm, whereas the corresponding standard error in the

TABLE XXIII

Statistics comparing the observed precipitable water values with those obtained by estimation of the λ-parameters for each sounding.

STATION	MEAN W (cm)			STANDARD DEVIATION, σ (W)			CORRELATION COEFFICIENT		R^2		STANDARD ERROR (cm)	
	W_o	W_{cs}	W_{cm}	σ_o	σ_{cs}	σ_{cm}	R_{cs}	R_{cm}	R^2_{cs}	R^2_{cm}	S_{cs}	S_{cm}
Waycross	1.2928	1.2828	1.2087	0.9425	0.9397	0.7951	.9931	.8207	.9863	.6735	.1142	.5574
San Diego	.9577	1.0112	1.0556	.2734	.2644	.4391	.9749	.6118	.9504	.3743	.0630	.2238
Spokane	.4957	.4884	.5141	.1879	.2123	.3031	.9711	.8551	.9431	.7312	.0464	.1008
Sault Ste. Marie	.4682	.4620	.4698	.2463	.2410	.2548	.9962	.9049	.9923	.8188	.0223	.1085
Column Means									.9680	.6495	.0701	.3093

comparison of W_{cs} with W_o is close to 0.31 cms. This larger error arises primarily from the southerly stations in Table XXIII.

In conclusion, the value of W_{cs} based upon a best-fit λ-value in conjunction with Eq. (13) gives a good estimation of W_o. On the other hand, the computation of W_{cm} wherein the λ-value is estimated by a multiple regression formula using gross parameters from the sounding has some skill in estimation of W_o, but perhaps needs additional study for selection of input variables for optimal inclusion in the multiple regression equation used to determine λ.

VI. CONCLUSIONS

At a group of 12 geographically diverse stations, the mixing-ratio profile was parameterized as a power-law profile, Eq. (6), having the characteristic exponent λ . For each sounding λ was derived by a least-squares fit with an accuracy usually characterized by a correlation coefficient of 0.95 or higher. At each station, the fit was most accurate in this form when the standard deviation of observed mixing-ratio was large. The set of λ -values were grouped together in time series at each station, and they were found to be quite variable in general. This variability of λ was related to the detailed structure of the moisture profile from the sounding data. A sizeable part of the standard deviation $\sigma_t(\lambda)$ was found to be explainable by use of a multiple regression procedure which incorporated easily defined properties of the sounding. The primary test of the λ -profile parameterization was made in application to the comparison of observed versus computed values of the precipitable water vapor at four stations, with the latter given by

$$W = \frac{(W_{500})(500)}{g(\lambda + 1)} \left[\left(\frac{P_s}{500}\right)^{\lambda+1} - \left(\frac{P_{top}}{500}\right)^{\lambda+1} \right] \tag{13}$$

On an individual sounding basis the λ -value gave good verification of the observed precipitable water vapor value. Over a period of time it was possible to generate multiple regression equations for λ which gave moderately good verification when applied for the appropriate sounding using Eq. (13). There is some possibility that this procedure can be employed in a predictive sense provided the parameters used in the regression can be successfully generated by a primitive equations model.

It is possible that the multiple regression sounding parameters may be selected to yield closer agreement with the λ -values determined from the individual soundings.

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An investigation is made into the accuracy of the description of the actual mixing-ratio profile by a power-law approximation. This approximation is obtained by a least-squares technique in order to derive the best-fit exponent λ of the pressure profile. The λ-values were computed for soundings at 12 geographically diverse stations over the United States during the period 16 - 23 March, 1971. The λ-values at each station were found to undergo a time-variation based mainly on the synoptic-scale

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variations in the mixing-ratio profiles. A stepwise multiple regression procedure involving up to four variables from the temperature-humidity soundings was utilized in order to "predict" the value of λ from gross-parameters of the soundings by station. Tests were performed at four stations in comparing the observed precipitable water vapor W_o with that computed from the power-profile W_{cs} which depended upon λ computed from the sounding. The values of W_{cs} gave a high correlation, $R(W_{cs}, W_o) > 0.97$, for each of the four stations. However, the values of W using λ from the predictive multiple regression equation gave somewhat larger standard errors at each of the four stations. The W_{cm}-values when compared to W_o indicated some measure of predictive skill.

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